# Vacation Travel Problem

**Suggested Grade/s:** 8th or 9th Grade Pre-Algebra or Algebra

**SD Mathematics Content Standard Strand:** Algebra
**SD Mathematics Content Standard**: 9-12.A.4.1 Students are able to use graphs, tables, and equations to represent linear functions.

**Task Summary:**
Students demonstrate their understanding of the graphing of linear functions to compare rental car rates and determine the most economical rental choice for a family vacation.

**Materials Needed**: Paper, Pencil, Graph Paper, Graphing Calculator

**Time and Context of Task**:
1-2 class periods. This task is for use upon completion of the study of slope and graphing linear equations/functions. It can be completed as a group activity or an individual activity. If students work alone, this task allows a teacher to assess individual progress. If more detail is desired in terms of a student presentation and/or delivery, then more time may be needed.

**Author for This Task:**
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## Performance Task

A family that flies into Sioux Falls from Phoenix, Arizona is planning a 1 week vacation in South Dakota and needs to rent a car. They researched and found the following options available in Sioux Falls:

*****Weekly Rate #1 $324/week, unlimited mileage
*****Weekly Rate #2 $210/week plus 12 cents per mile
*****partial week charged at full week price
******Daily Rate #1 $50/day, unlimited mileage
******Daily Rate #2 $42/day plus 3 cents per mile
******partial days charged at full day price

The family doesn't know exactly how far they will drive but estimate that it will be between 800 and 1050 miles. They must decide which plan to choose. Explore the four options below by first completing the table below.

**Comparison of Total Rental Car Costs Per Week Based on Mileage Driven**

| Total Miles Driven in One Week | 800 | 850 | 900 | 950 | 1000 | 1050 |
|---|---|---|---|---|---|---|
| Cost at Weekly Rate #1 | | | | | | |
| Cost at Weekly Rate #2 | | | | | | |
| Cost at Daily Rate #1 | | | | | | |
| Cost at Daily Rate #2 | | | | | | |

a) From this table, draw and compare the graphs of the four options on the same graph.

b) Analyze the graphs. Questions to consider: Is it appropriate to connect points on the graphs to make lines? Explain why or why not. Do all of the points of each graph lie on a straight line? What is a function called that has a graph which is a straight line? Which option increases the fastest? What is it's slope? Which option increases the slowest? What is it's slope? What is significant about points where graphs intersect?

c) Write the Total Week's Rental car cost as a function of the Number of Miles Driven for each option.

d) Based on the best economics, prepare a presentation that would explain under what conditions the family should choose each option.

## Content Standards

**Primary Standard for the Task:**
**Strand Name:** Algebra
**SD Goal:** Students will use the language of algebra to explore, describe, represent, and analyze number expressions and relations that represent variable quantities.
**Indicator:** Describe and use properties and behaviors of relations, functions, and inverses.
**Standard**: 9-12.A.4.1 Students are able to use graphs, tables, and equations to represent linear functions.

**Supplemental/Additional Standard for the Task:**
**Strand Name:** Algebra
**SD Goal:** Students will use the language of algebra to explore, describe, represent, and analyze number expressions and relations that represent variable quantities.
**Indicator:** Interpret and develop mathematical models
**Standard:** 9-12.A.3.1 Students are able to create linear models to represent problem situations.

**NCTM Process Standard:**
**Communicatio**n: Use the language of mathematics to express mathematical ideas precisely.
**Communication:** Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
**Connections**: Recognize and apply mathematics in contexts outside of mathematics.

**Problem-Solving Strategies:**

- Developing formulas and writing equations
- Drawing pictures, graphs, and tables
- Simplifying the problem

## Assessment Tools

### Task Rubric

| | **Advanced** | **Proficient** | **Basic** | **Below basic** |
|---|---|---|---|---|
| **9-12.A.4.1** Students are able to use graphs, tables, and equations to represent linear functions. | Draws and justifies valid and precise conclusions for each rental option.<br><br>Student is able to solve a system of linear equations to find a point where two plans will cost the same. | Draws and justifies valid conclusions for two or three rental options.<br><br>Student is able to create a linear model relating to each rate plan and is able to interpret the meaning of having two graphs intersect. | Draws and justifies valid conclusions for one rental option.<br><br>Student is able to graph a line for each rate plan using a table of values. | Draws no conclusion or draws an invalid conclusion.<br><br>Student is unable to graph a line for each rate using a table of values or is unable to complete the table of values comparing each rate plan. |
| Selection of the Type of Graphical Representation. | Displays the rental cost calculations in an appropriate graph with strong visual appeal. | Chooses to display the rental cost calculations in two appropriate graphs. | Chooses to display the rental cost calculations in more than two appropriate graphs. | Chooses an inappropriate graphical form or provides no graph. |
| Correctness of Weekly Rental Costs | Correctly calculates the rental cost for each rental option. | Correctly calculates the rental cost for most of the rental options. | Some inaccuracies in the calculation of the rental cost for each rental option. | Fails to calculate rental cost for each rental option or has gross misunderstandings. |
| Correctness of Weekly Rental Costs Written as Linear Functions | All rental options are written correctly as linear functions. | The majority of the rental options are written correctly as linear functions. | Some evidence of making the connection that each rental option could be written as a linear function. | No evidence of linear function understanding. |
| Communicate Mathematically | Clearly and consistently uses language that is mathematically correct. | Uses clear language that frequently includes appropriate mathematical terminology. | Uses language that sometimes is mathematically correct. | Uses vague language that does not use mathematical terminology. |
| Convincing Presentation | Presentation shows complete understanding of the mathematical concepts used. It is organized, clear, and convincing. | Presentation shows substantial understanding of the mathematical concepts used. Some organization but not very convincing. | Presentation shows some understanding of the mathematical concepts used. Very little organization. Conclusions are not convincing. | Presentation shows very limited understanding of the underlying concepts needed or no attempt to convince. |

**Eighth Grade Algebra**
**Performance Descriptors**

| | |
|---|---|
| **Advanced** | **Eighth grade students performing at the advanced level:**<br>• represent using 1st degree algebraic statements using integers, tables, and graphs, in order to justify solution(s). |
| **Proficient** | **Eighth grade students performing at the proficient level:**<br>• simulate situations using 1st degree algebraic statements using integers, tables, and graphs in order to determine solution(s). |
| **Basic** | **Eighth grade students performing at the basic level:**<br>• simplify, solve, and graph 1st degree algebraic statements using whole numbers. |

**Eighth Grade Algebra**
**ELL Performance Descriptors**

| | |
|---|---|
| **Proficient** | **Eighth grade ELL students performing at the proficient level:**<br>• solve algebraic equations involving rational numbers;<br>• use tables and graphs to justify solutions;<br>• read, write, and speak the basic language of algebra. |
| **Intermediate** | **Eighth grade ELL students performing at the intermediate level:**<br>• solve algebraic equations involving integers;<br>• use tables and graphs to determine solutions verbally or in writing;<br>• create numerical expressions from oral or written contexts;<br>• explain in mathematical terms the sequence of steps used in solving problems;<br>• given simple oral or written responses to directed questions on topics presented in class. |
| **Basic** | **Eighth grade ELL students performing at the basic level:**<br>• evaluate numerical expressions using integers;<br>• read tables and graphs;<br>• recognize and use basic algebraic terms;<br>• respond to yes or no questions and to problems presented pictorially or numerically in class. |
| **Emergent** | **Eighth grade ELL students performing at the emergent level:**<br>• respond to numerical (not word) problems using addition, subtraction, multiplication, and division;<br>• use a number line to solve simple problems involving integers;<br>• copy and write numerals and algebraic symbols;<br>• imitate pronunciation of numbers and mathematical terms;<br>• use non-verbal communication to express mathematical ideas. |
| **Pre-emergent** | **Eighth grade ELL students performing at the pre-emergent level:**<br>• observe and model appropriate cultural and learning behaviors from peers and adults;<br>• listen to and observe comprehensible instruction and communicate understanding non-verbally. |

**Core High School Algebra**
**Performance Descriptors**

| | |
|---|---|
| **Advanced** | **High school students performing at the advanced level:**<br>• transform algebraic expressions;<br>• solve quadratic equations;<br>• solve a system of linear equations. |
| **Proficient** | **High school students performing at the proficient level:**<br>• transform polynomial expressions using real number properties;<br>• solve single variable linear equations with integral coefficients;<br>• graph linear equations;<br>• interpret tables, graphs, and charts to solve problems;<br>• create a linear model from a problem context. |
| **Basic** | **High school students performing at the basic level:**<br>• transform linear expressions with integral coefficients using real number properties;<br>• solve linear equations of the form $ax + b = c$, where a, b, and c are integers;<br>• recognize the graph of a linear equation;<br>• graph a line from a table of values. |

**Core High School Algebra**
**ELL Performance Descriptors**

| | |
|---|---|
| **Proficient** | **High school ELL students performing at the proficient level:**<br>• solve, transform, and graph linear equations;<br>• apply algebraic representations to solve problems;<br>• read, write, and speak the language of algebra and apply it to algebraic problem-solving situations. |
| **Intermediate** | **High school ELL students performing at the intermediate level:**<br>• solve one-variable linear equations;<br>• graph linear equations in slope-intercept form;<br>• complete tables to graph linear equations;<br>• create numerical expressions from oral or written contexts;<br>• evaluate an algebraic expression given the value of the variable(s);<br>• explain in algebraic terms the steps and/or strategies used in problem solving;<br>• give oral, pictorial, symbolic (diagrams) or written responses to questions on topics presented in class. |
| **Basic** | **High school ELL students performing at the basic level:**<br>• graph points on a coordinate system;<br>• solve problems with integral and rational solutions;<br>• evaluate numerical expressions;<br>• demonstrate problem-solving strategies;<br>• break tasks into smaller parts and make connections to prior knowledge;<br>• recognize, compare, and use appropriate algebraic terms;<br>• respond to yes or no questions and to problems presented pictorially or numerically in class. |
| **Emergent** | **High school ELL students performing at the emergent level:**<br>• identify and use mathematical symbols;<br>• copy and write numerals and algebraic symbols;<br>• imitate pronunciation of numerals and mathematical terms;<br>• use non-verbal communication to express mathematical ideas. |
| **Pre-emergent** | **High school ELL students performing at the pre-emergent level:**<br>• observe and model appropriate cultural and learning behaviors from peers and adults;<br>• listen to and observe comprehensible instruction and communicate understanding non-verbally. |

**Student Work Sample:**

**Comparison of Total Rental Car Costs Per Week Based on Mileage Driven**

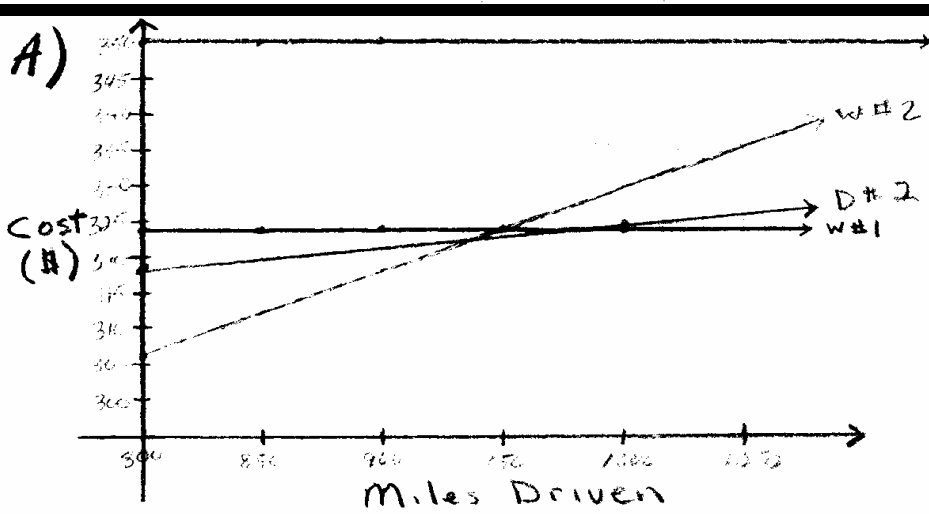
| Total Miles Driven in One Week | 800 | 850 | 900 | 950 | 1000 | 1050 |
|---|---|---|---|---|---|---|
| Cost at Weekly Rate #1 | $324 | $324 | $324 | $324 | $324 | $324 |
| Cost at Weekly Rate #2 | $306 | $312 | $318 | $324 | $330 | $336 |
| Cost at Daily Rate #1 | $350 | $350 | $350 | $350 | $350 | $350 |
| Cost at Daily Rate #2 | $318 | $319.50 | $321 | $322.50 | $324 | $325.50 |

$\frac{6}{50} = \frac{3}{25}$

$\frac{1.5}{50}$

Austin

A)




B) Connecting points to graph a line shows the cost for any amount of mileage driven. Each graph represents a linear function. Slopes are constant. The slope for D#1 and W#1 is zero since there is no increase in cost for more miles driven. W#2 has a slope of $\frac{6}{50} = \frac{3}{25}$. D#2 has a slope of $\frac{1.5}{50} = \frac{3}{100}$.

W#2 plan increases the fastest. D#2 plan increases more slowly than W#2.

Where lines intersect, rate plans cost the same for the same amount of miles driven.

C) Let y = cost in $
x = # miles driven

Weekly Rate #1 $y = 324$
Weekly Rate #2 $y = \frac{3}{25}x + 210$ $\begin{cases} y = mx+b \\ 306 = \frac{3}{25}(300)+b \\ 306 = 96+b \\ 210 = b \end{cases}$
Daily Rate #1 $y = 350$
Daily Rate #2 $y = \frac{3}{100}x + 294$ $\begin{cases} y = mx+b \\ 318 = \frac{3}{100}(800)+b \\ 318 = 24+b \\ 294 = b \end{cases}$

D) (other sheet)

W#2 = D#2 (by substitution)

$\frac{3}{25}x + 210 = \frac{3}{100}x + 294$

$\frac{9}{100}x = 84$

$x = \frac{2800}{3}$ or $933\frac{1}{3}$ miles

**Comparison of Rental Plans**

**Weekly Rate #1**
This plan according to the graph would be the best if the family would end up driving more than 1000 miles. At exactly 1000 miles, the daily rate #2 plan and the weekly rate #1 plan cost the same.

**Weekly Rate #2**
This plan will be the cheapest of all rates if the family plans to drive fewer than 933 1/3 miles. This was determined by setting the equations in part c equal to each other and solving for x. The cost for this amount of mileage would be $322. For the same mileage, it would cost the same as the daily rate #2 plan. Once the family travels more than 950 miles this plan is no longer a good option because two other plans are cheaper.

**Daily Rate #1**
This plan is the worst plan of the four. It should not even be considered

**Daily Rate #2**
If the family plans to drive between 933 1/3 miles and 1000 miles this plan would be the best value.

**Overall**
If it were left up to me I would choose the weekly #1 plan. It is the best value if the number of miles traveled were at least 1000 miles. A person never knows if road construction encountered along the way would require a long detour. Using the graph, if money could be an issue, my secondary choice would be the daily #2 plan because its increase is a gradual one.

**Looking at Student Work – Instructor notes and rating for work sample:**

Based on the rubric for this performance task I would rate this student as being advanced. The student achieves all criteria in the advanced column of the rubric.

## Instructional Notes

**Author comments**

To get student samples for this project in a timely manner, this activity was given in the fall to geometry students who had just completed algebra last spring. This task could be used in an algebra class after studying linear equations and/or systems of linear equations.

**Task Extensions**

Have students write their own rate plan problem. Calling card and cellphone rate plan comparisons are other real life sources of information that are fun to discuss with students.

**Common Strategies that students use to successfully complete the performance task:**

Using graph paper to display all four rate plans helped students move along quickly and helped them make connections within the task. (points – lines – slope – linear equations/functions – systems)

**Common Misunderstandings that students exhibited while attempting to complete the performance task:**

A few students attempted to graph the daily rate plans and weekly plans on separate graphs even though the directions asked them to graph all four options on the same graph. This made it more difficult to compare rate plans. Since this task was given to students not currently enrolled in algebra, some mistakes were made such as forgetting what a linear function was, what the slope-intercept form of a linear equation looked like, and calculating slope as the change in x divided by the change in y.

**Appropriate Technology for This Lesson**:

Graphing Calculator
TI-Connect Software

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## Instructional Resources

**SD Mathematics Content Standards**
http://www.doe.sd.gov/contentstandards/math/index.asp

**SD Assessment and Testing**
http://www.doe.sd.gov/octa/assessment/index.asp

**The National Assessment of Educational Progress (NAEP)**
http://www.doe.sd.gov/octa/assessment/naep/index.asp

**National Council of Teachers of Mathematics**
http://nctm.org/

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